

Optimisation of machining parameters for hard machining: grey relational theory approach and ANOVA

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Abstract The present paper deals with experimental investigations carried out for machinability study of hardened steel and to obtain optimum process parameters by grey relational analysis. An orthogonal array, grey relations, grey relational coefficients and analysis of variance (ANOVA) are applied to study the performance characteristics of machining process parameters such as cutting speed, feed, depth of cut and width of cut with consideration of multiple responses, i.e. volume of material removed, surface finish, tool wear and tool life. Tool wear patterns are measured using optical microscope and analysed using scanning electron microscope and X-ray diffraction technique. Chipping and adhesion are main causes of wear. The optimum process parameters are calculated for rough machining and finish machining using grey theory and results are compared with ANOVA.

Keywords Grey relational analysis · Hard machining · End milling

1 Introduction

It is important to choose the best machining parameters for achieving optimum performance characteristics for hard machining, which comprises rough and finish machining. The desired machining parameters are usually selected with

the help of referred handbooks, past experience and various trails. However, the selected machining parameters may not be optimal or near optimal machining parameters. Taguchi method can be applied for optimisation of process parameters to produce high quality products with lower manufacturing costs [1]. Taguchi's parameter design is one of the important tools for robust design, which offers a systematic approach for parameters optimisation in terms of performance, quality and cost [1–5]. Taguchi technique had been applied to optimise machining process parameters for end milling process of hardened steel AISI H13 machined with TiN-coated P10 carbide insert [3]. The same methodology had been used by Nalbant et al. [4] to find the optimal cutting parameters, i.e. insert radius, feed rate and depth of cut for surface roughness in turning operation based on experimental results done on AISI 1030 steel bars using TiN-coated tools. Further, design optimisation for quality was carried out by Yang and Tarn [5] to find the optimal cutting parameters in turning process for S45C steel bars using tungsten carbide cutting tools by orthogonal array and analysis of variance. Thus, Taguchi methodology can be effectively used to optimise process parameters for single performance characteristic only. However, the optimisations of multiple performance characteristics find more applications and it is also an interesting research programme. Grey theory can provide an efficient solution to the uncertainty in multi-input and discrete data problems. It had been effectively applied to optimise the multi-response processes through the setting of process parameters [6–10]. It is an effective method to analyse the relational degree between discrete sequences. The advantage of the above method is that many factors can be analysed using less data [8]. Grey relational analysis can be used to find out the relationship of the reference sequence with other sequences or the relational degree existing

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between the variations of any two different sequences. The approach about grey theory and its applications are illustrated in [11–17]. Basic introduction about analysis of variance (ANOVA) and its applications are described in [18,19]. The process parameter optimisation using ANOVA for milling and hard turning processes are analysed and well represented in [20–22]. Machinability study for tool life and surface roughness was investigated on AISI D2 steels (58 HRC) by Koshy et al. [23] using indexable ball nose end mills employing carbide, cermet tools and solid carbide ball nose end mills. It was observed through tool wear study that chipping, adhesion and attrition were principal mechanisms responsible for tool wear. Machining investigations of the silver toughened alumina inserts were done by Dutta et al. [24]. Abrasion and plastic deformation were considered to be active wear mechanisms for the aforementioned inserts. High speed milling of hardened steel with a comprehensive review comprising process parameters, tool life and work piece surface roughness had been described in [25–31]. Machining investigations on machining of AISI D2 steel of hardness 62 HRC with polycrystalline cubic boron nitride (PCBN) tool inserts were carried out and it had been observed that tool inserts failed by flank wear and most feasible feeds and speeds fell in the range of 0.08–0.20 mm/rev and 70–120 m/min, respectively [25]. Investigations on martensitic stainless steel (60 HRC) using alumina-based ceramic cutting tools and various types of wear were observed that flank wear could affect the tool life at lower speed; however, crater wear could affect tool life at high speed, i.e. above 200 m/min [26]. Performance investigations were carried out using minimum quantity lubricant (MQL) for turning process. It had been investigated that when MQL applied to the tool rake face, the tool life was not enhanced. But when MQL applied on the flank face, the tool life was increased [27]. However, it was not easy to retain lubricant at the flank side. Investigations were carried out by end milling process on AISI D3 cold-work tool steel hardened to 35 HRC by Camuscu and Aslan [28] using coated carbide, coated cermet, alumina (Al_2O_3)-based mixed ceramic and CBN inserts. The cutting tool performances were compared with respect to tool life and surface finish of the workpiece. From the investigations, it has been observed that CBN tool yielded the best performance than other cutting tools. Experimental investigations of cemented carbide chamfered tools during continuous and interrupted turning of medium carbon low alloy steel were done for cutting force, tool life and chip formation by Choudhury et al. [29]. It has been observed that both in continuous and interrupted turning, with the increase in chamfer width, both the main cutting force and feed forces did increase and the effect on the feed force was more significant. The performance of chromium carbide-coated carbide tool inserts and micron drills in dry

machining was investigated by Su et al. [30]. It had been observed that chromium carbide-coated inserts (Cr10%C) showed the best wear resistance in AISI 1045 steel turning test and inserts (Cr50%C) performed exceptionally well. The machinability of ultra-high speed milling of hardened steels was reviewed by Dewas and Aspinwall [31] for tool life, workpiece surface finish/dimensional accuracy and cost data. Promising results had been demonstrated when milling a range of hardened tool steels using machining centres equipped with high speed spindles and cutting tools, i.e. cemented tungsten carbide, cermet, conventional ceramics and PCBN inserts. The effect of machining parameters, tool wear on chip morphology and surface integrity during high speed machining of D2 tool steel were investigated theoretically and experimentally with PCBN inserts [32,33]. The effect of edge preparation in CBN cutting tools was investigated on process parameters and tool performance. The finite element simulations and high speed orthogonal cutting tests were carried out to optimise tool life and surface finish in hard machining of AISI H-13 hot work tool steel [34]. Performance investigation and wear mechanism of a binderless PCBN tool in high speed milling of grey cast iron was explored by Kato et al. and concluded that binderless PCBN could provide longer tool life and excellent surface finish [35].

It has been observed from the literature survey that Taguchi methodology can be applied for analysing the best process parameters for single performance characteristics only, i.e. tool wear, tool life, surface finish, volume of material removal, one parameter at a time only, where as grey relational analysis can effectively be used for analysing multi-performance characteristics incorporating the above all parameters at a time. Further, from the experimental investigations done on hardened steels using various cutting tools including CBN, it can be observed that carbide cutting tools are widely used in tool and die making industries despite the better performance by CBN inserts. It has been observed that the use of CBN inserts may not be economically viable as these are very costly.

The main objectives are to study machinability aspects, to apply grey theory and ANOVA for selecting the best process parameters whilst machining of IMPAX HI HARD tool steel for both rough and finish machining operations. In the present investigations, coated carbide inserts are used for rough machining operations and ball end mill cutters are used for finish machining operations.

2 Grey relational analysis

In this paper, Taguchi L_{18} orthogonal array is integrated with grey relational theory to analyse the process parameters obtained from 18 experiments for rough and finish

machining individually by varying four process parameters, i.e. cutting speed, feed, depth of cut and width of cut. For rough machining, the process parameters are optimised with respect to volume of material removed, tool wear and tool life (cut length) by using grey relational analysis. The most influencing parameters are noticed. Similarly, the process parameters are optimised for finish machining with respect to surface finish, tool wear and tool life (cut length) and the influencing parameters are noticed.

The optimisation of inter-related multi-performance characteristics using grey system theory based on Taguchi orthogonal array experimental data has been developed by Deng [8,9]. In grey relational analysis, the first step is data pre-processing. This avoids the problem of different scales, units and targets. The “Appendix” shows a worked example. Experimental data are normalised in the range between zero and one. Next, the grey relational coefficient is calculated from the normalised experimental data to express the relationship between the ideal (best) and the actual experimental data. Grey relational grade is then computed by averaging the grey relational coefficients corresponding to each performance characteristic. The experimental data of the multi-response characteristics is evaluated by using this grey relational grade. The optimum level of the process parameters is the level with the highest grey relational grade.

The data sequence for volume of material removed, tool life, i.e. cut length, which are higher-the-better performance characteristic, are pre-processed as follows:

$$x_i^*(k) = \frac{x_i^0(k) - \min x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \quad (1)$$

where $k=1$ to n , $i=1$ to 18, n is the performance characteristic and i is the trial number.

Tool wear and surface roughness, which are lower-the-better performance characteristic, are pre-processed as follows:

$$x_i^*(k) = \frac{\max x_i^0(k) - x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \quad (2)$$

where,

$x_i^*(k)$ is the value after grey relational generation
 $\min x_i^0(k)$ is the smallest value of $x_i^0(k)$
 $\max x_i^0(k)$ is the largest value of $x_i^0(k)$

The experimental and normalised results for volume of material removed, tool wear and tool life are tabulated for rough machining. Similarly, for finish machining, the experimental and normalised results for surface roughness, tool wear and tool life are tabulated. The higher pre-processed value shows better performance and best pre-processed result should be equal to one.

The grey relational coefficient $[\xi_i(k)]$ can be calculated as follows:

$$\xi_i(k) = \frac{\Delta_{\min} + \varsigma \Delta_{\max}}{\Delta_{0i}(k) + \varsigma \Delta_{\max}} \quad (3)$$

“Note that higher is better and is achieved when $X_i(k)=X_0(k)$, i.e. when X = reference” where,

$x_0^*(k)$ denotes the reference sequence
 $x_j^*(k)$ denotes the comparability sequence
 $\varsigma \in [0-1]$ is the distinguishing coefficient; 0.5 is widely accepted
 $\Delta_{0i} = \|x_0^*(k) - x_i^*(k)\|$ is the difference in absolute value between $x_0^*(k)$ and $x_i^*(k)$
 $\Delta_{\min} = \min_{\forall j \in i} \min_{\forall k} \|x_0^*(k) - x_j^*(k)\|$ is the smallest value of Δ_{0i}
 $\Delta_{\max} = \max_{\forall j \in i} \max_{\forall k} \|x_0^*(k) - x_j^*(k)\|$ is the largest value of Δ_{0i}

After calculating grey relational coefficients, the grey relational grade is obtained as:

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n \xi_i(k) \quad (4)$$

where γ_i is the grey relational grade and n is the number of performance characteristics.

The grey relational coefficients and corresponding grey relational grade for each experiment for rough and finish machining are calculated. The higher value of grey relational grade is near to the product quality for optimum process parameters.

3 Analysis of variance

The influence of any given input parameter for the machining process can be determined by ANOVA from a series of experimental results by the design of experiment approach. ANOVA was first developed by British Statistician Ronald A Fisher and the concepts are described in [18,19]. This technique can be applied for process parameter optimisation of machining parameters for milling and turning processes [20–22]. It is the predominant statistical method used to interpret experimental data. It is

designed to represent a concept that any high-dimensional function may be broken down into a subset of terms from the expansion:

$$f(x) = f_0 + \sum_{i=1}^p f_i(x_i) + \sum_{i=1}^p \sum_{j=i+1}^p f_{ij}(x_i, x_j) + f_{1,2,\dots,p}(x) \quad (5)$$

where p represents the number of inputs, f_0 is a constant (bias term) and the other terms on the right hand side represent the univariate, bivariate, trivariate etc., functional combinations of the input parameters.

ANOVA is a mathematical technique, which partitions the total variation into its appropriate components. Thus, the total variation of the system, defined by the total sum of squares term:

$$SS_T = \sum y_i^2 \text{ for } i = 1, 2, \dots, p, \quad (6)$$

can be given as:

$$SS_T = SS_m + SS_e \quad (7)$$

where $SS_m = pM^2$ and $SS_e = \sum (y_i - M)^2$ are the mean sum of squares and the error sum of squares, respectively, with $M = 1/p \sum y_i$ ($i=1, 2, \dots, p$). In the case of two-way ANOVA, when the interaction effect of main factors affects the output values, the total variation may be decomposed into more components:

$$SS_T = SS_A + SS_B + SS_{AB} + SS_e \quad (8)$$

where SS_A and SS_B represent variations due to the factors A and B , respectively, whilst $SS_{AB} = \sum (AB)_i^2 / p_{ABi}$ for $i=1, 2, \dots, k$ is variation due to the interaction of factors A and B , where k represents the number of possible combinations of interacting factors and p_{ABi} is the number of data points under this condition.

Degrees of freedom need also be considered together with each sum of squares whilst performing ANOVA calculations. The determination of error variance is an essential step as ANOVA involves experimental studies with certain test error. As the sample size establishes the confidence level of the results, the sample variance within the factor levels should be calculated. Subsequently, the obtained data are used to estimate the value F of the Fisher test (F test). The total variation observed in an experiment attributed to each significant factor and/or interaction is reflected in the percent contribution (P). This shows the relative power of a factor and/or interaction to reduce variation. The factors and interactions with substantial percent contribution are the most important.

4 Experimental works

4.1 Work piece

Work piece is pre-annealed tool steel with a hardness of 55 HRC. The chromium, nickel and manganese alloyed material offers a very good polishability and photo etching properties. It is used for mould making applications through the manufacturing process, which ensures high purity, good homogeneity and uniform hardness. The material compositions are shown in Table 1.

4.2 Design of experiments

The design of experiment has been used to analyse the effect of four machining process parameters for rough machining, i.e. cutting speed, feed, depth of cut and width of cut on three important output parameters such as volume of material removed, tool wear and tool life. The output parameters in the case of finish machining are surface finish, tool wear and tool life. The design of experiments using the orthogonal array is, in most cases, efficient when compared to many other statistical designs. The minimum number of experiments that are required to conduct the Taguchi method can be calculated based on the degrees of freedom approach. First, the degrees of freedom must be calculated before an orthogonal array is selected. It can be calculated by the following equation.

$$N = 1 + \sum_{i=1}^{\text{No of variables}} (L - 1) \quad (9)$$

where N is the number of degrees of freedom and L is the number of levels of machining input variables. Accordingly, Taguchi based L_{18} orthogonal array is selected. This is shown as experimental layout in Table 2. Eighteen experiments were carried out accordingly to study the effect of above machining input parameters. Each experiment was repeated three times in order to eliminate the experimental error. The machining parameters and their levels are shown in Table 3.

4.3 Experimental procedure for rough machining

Experiments were performed on high performance Heyligenstaedt FH1 computer numerical control (CNC) milling

Table 1 Work piece material composition

Element	C	Cr	Mn	Si	Ni	Mo	S
% by wt	0.37	2.0	1.4	0.3	1.0	0.2	0

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Table 2 Experimental layout using L_{18} orthogonal array

Experiment no.	End milling machining parameters			
	Cutting speed	Feed	Depth of cut	Width of cut
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	1	2
5	2	2	2	3
6	2	3	3	1
7	3	1	2	1
8	3	2	3	2
9	3	3	1	3
10	1	1	3	3
11	1	2	1	1
12	1	3	2	2
13	2	1	2	3
14	2	2	3	1
15	2	3	1	2
16	3	1	3	2
17	3	2	1	3
18	3	3	2	1

machine. The working space X , Y and Z movements are $1,550 \times 880 \times 550$ mm with variable spindle speeds (maximum 30,000 rpm). The main spindle power is 75 kW and feed rate is 10 m/min. Tests were performed in two phases. In the first phase, rough machining was performed followed by finish machining. Rough machining was performed using tool holder diameter of 32 mm considering high material removal rate. The tool insert is of circular shape, physical vapour deposition (PVD)-coated carbide insert of TiAlN. The rake angle is 0° and the clearance angle is 11° . In all tests, flank wear/chipping was measured using optical microscope. Tests were stopped for rough machining when the maximum flank wear/chipping on the milling cutter reached 0.2 mm or breakage occurred. Subsequently, tool life in terms of cut length and volume of material removed was calculated. The volume of material removed is calculated by knowing cut length, depth of cut and width of cut. Tool wear patterns were analysed using scanning electron microscope (SEM) and X-ray diffraction technique. SEM pictures were taken. Chips were also collected and scanned for analysis.

4.4 Experimental procedure for finish machining

The surface finish is the best and desired parameter to evaluate the machinability of finish machining. Generally, ball end mill cutters are employed to achieve smooth and

high values of surface finish in three-axis machining, highest flexibility that suits the most of the applications in tool and die making industries. Experiments were performed on high performance Heyligenstaedt FH1 CNC milling machine using ball end mill cutter of diameter 10 mm. The rake, clearance and helix angles are -10° , 10° and 30° , respectively. Flank wear/chipping was measured using optical microscope. For finish machining, tests were stopped, when maximum flank wear/chipping reached 0.1 mm. The surface roughness was measured using perthometer. Work piece surface roughness and tool life in terms of cut length were investigated. Wear patterns were analysed using scanning electron microscope and X-ray diffraction technique.

5 Results and discussions

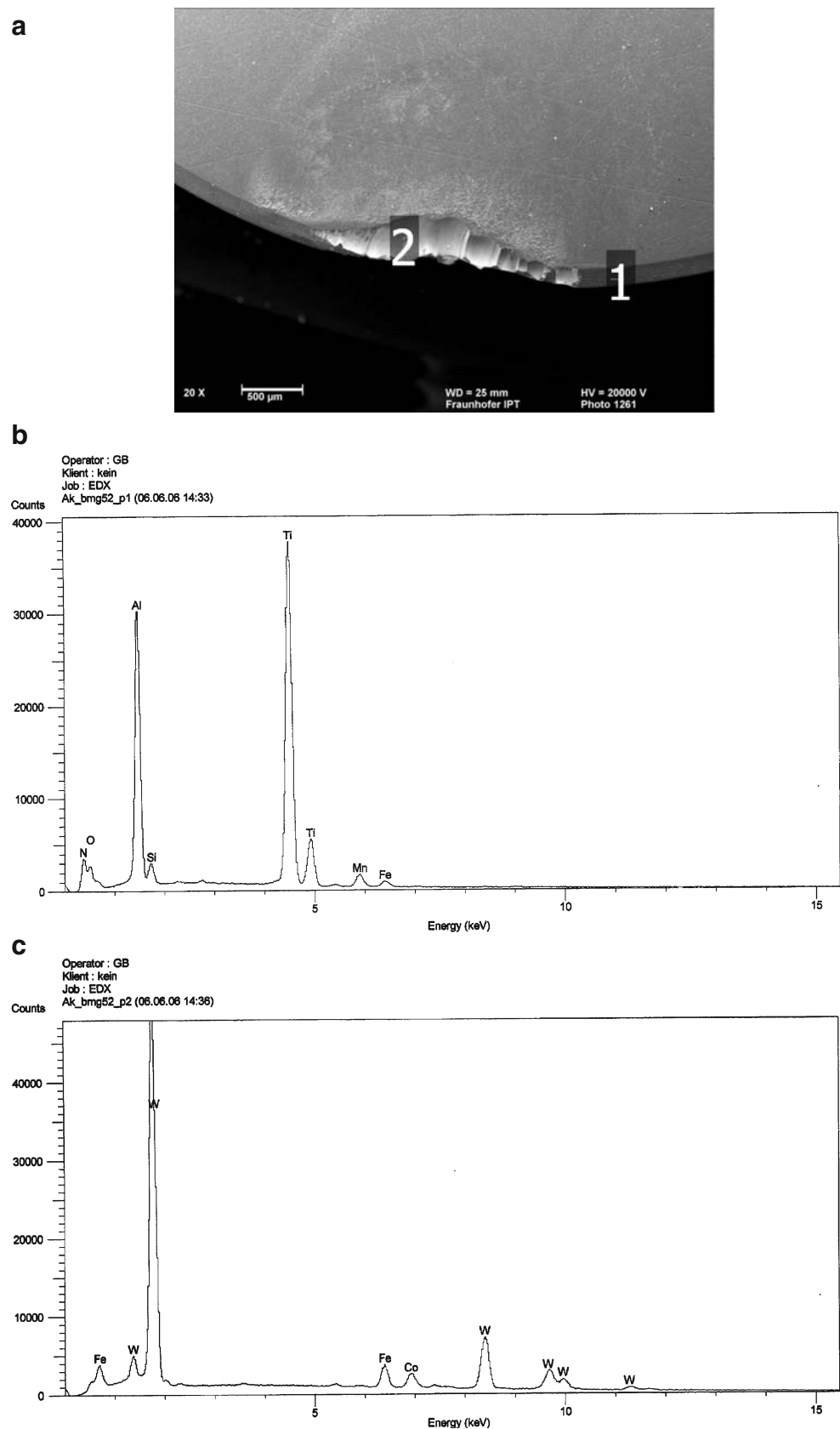
5.1 Rough machining

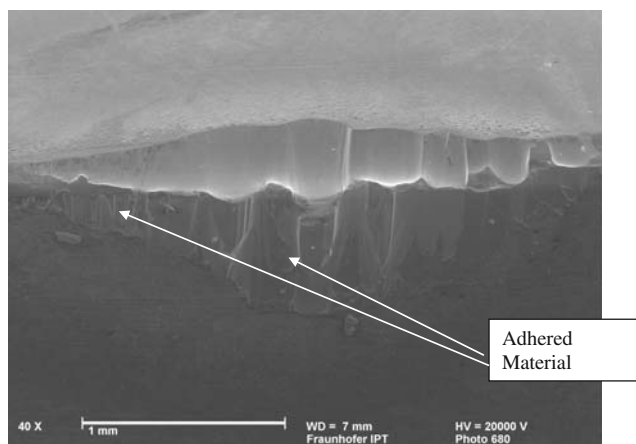
The cutting tool performance is evaluated mainly by tool life, as it is an important parameter. Tool wear is an important criterion whilst determining tool life as it affects dimensional tolerance and surface integrity of the work piece. Tool wear is characterised mainly by flank wear and its progressive growth [36]. Due to abrasion of work piece surface against the flank of cutting tool, the flank wear propagates with increase in cutting speed and rise in cutting temperature. When the average flank wear reaches a limit, i.e. 200 μm for rough machining using indexable cutter inserts and 100 μm for finish machining using ball end mill cutters, the tool insert needs to be replaced. It is perceived to be a failure. The rejection of tool, i.e. failure, is based on the following criteria as per ISO 8688-2 [37]: (a) The average flank wear or maximum flank wear, (b) crater wear depth and (c) flanking or fracture. The average or maximum flank wear is the limiting factor that controls tool life in the present study. During machining, cutting tool inserts/tools were withdrawn at regular intervals after some specific cut length, the nature, extent of wear failure and its cutting edge was observed and measured using optical microscope. Tool inserts were analysed for energy dispersive X-ray (EDX) diffraction process to observe for diffusion phenomenon from work piece to tool inserts. It

Table 3 Machining parameters and their levels (rough machining)

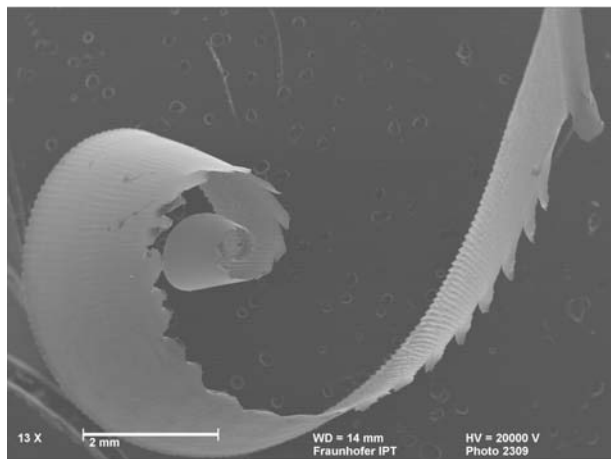
Machining parameters	Unit	Level 1	Level 2	Level 3
Cutting speed (A)	m/min	150	180	250
Feed (B)	mm/tooth	0.5	0.8	1.0
Depth of cut (C)	mm	0.5	0.8	1.0
Width of Cut (D)	mm	5	10	20

Fig. 1 **a** EDX picture of cutting tool insert at cutting speed 250 m/min, feed 1.0 mm/tooth, depth of cut 0.5 mm and width of cut 20 mm (tool life 12.3 m). **b** Composition of cutting tool insert at location 1. **c** Composition of cutting tool insert at location 2





(a) SEM picture of cutting tool insert



(b) SEM picture of chip

Fig. 2 Chip formation and tool wear at cutting speed 250 m/min, feed 0.8 mm/tooth, depth of cut 0.5 mm and width of cut 20 mm. **a** SEM picture of cutting tool insert. **b** SEM picture of chip

is observed from Fig. 1a–c that there is no diffusion phenomenon observed from work piece material into cutting tool insert. It is pointed out in Fig. 1a that point 1 is unworn tool insert surface, whereas point 2 is worn surface. The composition of aluminium, titanium and nitride is noticed in Fig. 1b which is nothing but the PVD coatings of TiAlN tool insert, whereas at point 2, the coating peeled off as observed in Fig. 1c but no diffusion process. Tool wear patterns were analysed using scanning electron microscope. These are shown in Fig. 2. The dominant wear pattern is observed to be non-uniform wear at the flank surface under all cutting conditions. The chipping and adhesion are primarily tool wear mechanisms whilst machining with coated carbide inserts. For a higher feed and depth of cut, the magnitude of cutting forces generated is higher. This rises the temperature causing the adhesion of work piece material on to the tool face (Fig. 2). Tool wear vs feed is shown in Fig. 3. It is noticed that the wear is almost constant as the cutting tool is withdrawn when the wear reaches a limit and subsequently the tool life is evaluated. The high cutting forces increase stresses in the contact region causing higher wear. The presence of high hardness chromium carbide content is responsible for imparting good wear resistance to the material (Table 1) which promotes attrition wear of tool inserts and renders the materials very difficult to machine. For end milling process, whilst the cutting edge engages the work piece material, the chip thickness does constantly change as orthogonal cutting does heavily differ from the engagement situation with discontinuous chipping. However, engagement of several cutting edges makes a process stable. It is observed from Fig. 4, the tool life is longer at lower cutting speed. However, when the cutting speed exceeds a limit, the tool life becomes shorter. The limiting value is 180 m/min. Experiments were also carried out for various feeds (fz) 0.5–1.0 mm/tooth but the results are yielding longer tool life at feed (fz) 0.5 mm/tooth. However, the tool life is shortened if

Fig. 3 Tool wear vs feed (rough machining)

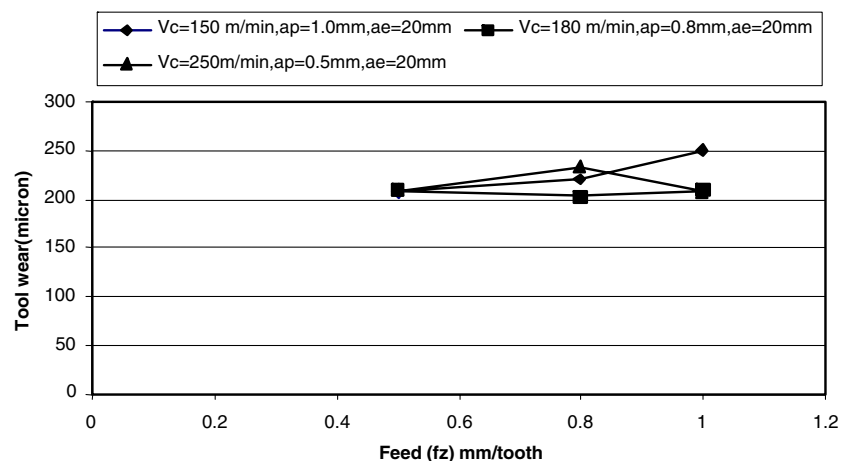
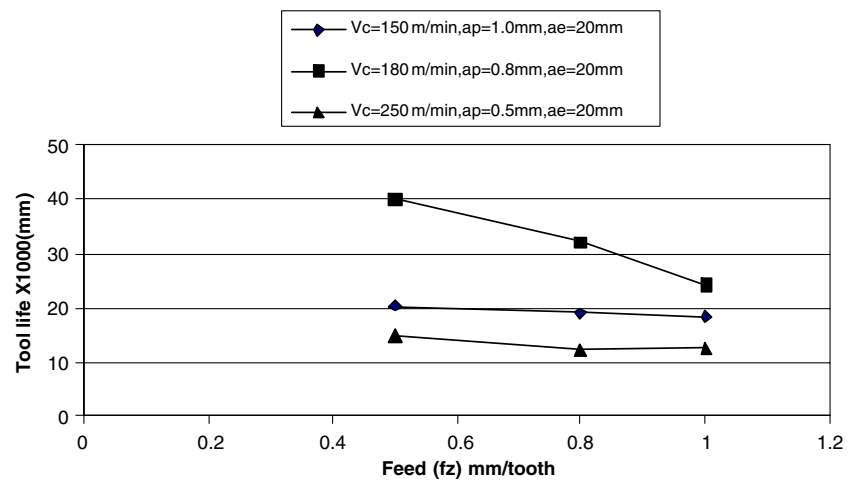


Fig. 4 Tool life vs feed (rough machining)

the feeds are lower than (fz) 0.5 mm/tooth. This is due to the inefficient removal of material by rubbing rather than cutting, resulting insufficient undeformed chip thickness. It is observed from Fig. 4 that the tool life decreases with increasing feed for a particular cutting speed. It is observed that the tool life is decreased by 40% when feed is increased from (fz) 0.5 to 1.0 mm/tooth for cutting parameters cutting speed (Vc) 180 m/min, depth of cut (ap) 0.8 mm and width of cut (ae) 20 mm. The volume of material removed (millimetre cube) for various feeds corresponding to cutting speed (Vc), depth of cut (ap) and width of cut (ae) is shown in Fig. 5. It is observed from Fig. 5 that the volume of material removal is more at feed (fz) 0.5 mm/tooth. At higher feeds, the tool life is shorter and hence, volume of material removal is less. When the feed is decreased, the tool life is longer which in turn gives more volume of material removal. It is observed from Fig. 5 that the volume of material removal is maximum ($630 \times 10^3 \text{ mm}^3$) at feed (fz) 0.5 mm/tooth corresponding to cutting parameters (Vc) 180 m/min, (ap) 0.8 mm and (ae) 20 mm. It is also observed that the

volume of material removed is increased by 40% when feed (fz) is decreased from 1.0 to 0.5 mm/tooth for cutting parameters (Vc) 180 m/min, (ap) 0.8 mm and (ae) 20 mm. In the milling process, the undeformed chip thickness varies from zero to feed per revolution per tooth during one cycle of cutter revolution. As the chip thickness changes, the cutting forces also vary. The stresses imposed on the cutting edge also fluctuate correspondingly which encourage flank wear and attrition wear. It has been pointed out by Oxley [38] that the size effect exists, when the undeformed chip thickness is less than 0.05 mm. It has been elaborated in [38,39] that the rate of increase of cutting force is less than that of the undeformed chip thickness. The specific cutting pressure acting on the tool–chip interface approaches a constant value [38], when the chip thickness at that instant is greater than 0.05 mm. Therefore, the effect of the specific cutting pressure on the tool–chip interface does not vary significantly with the feeds. However, at higher feeds, the average undeformed chip thickness is larger, resulting increase in radial cutting force and thus causes

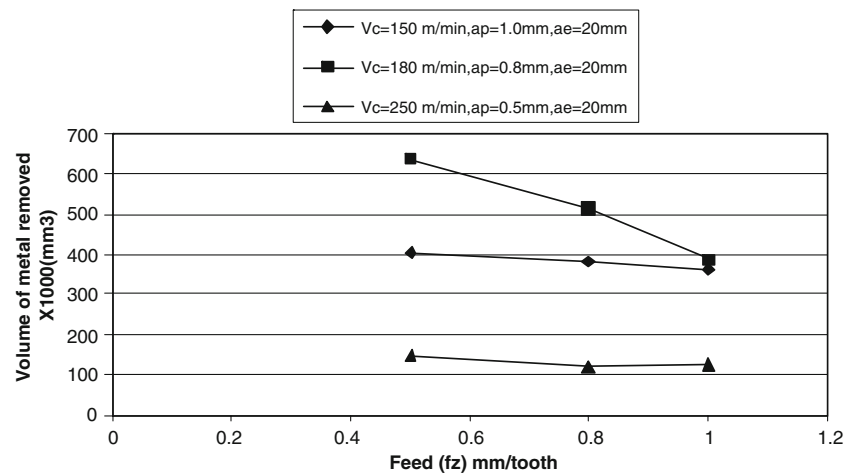
Fig. 5 Volume of material removed vs feed (rough machining)

Table 4 Experimental results for volume of material removed, tool wear and tool life (rough machining)

Experiment no.	Volume of material removed $\times 1,000$ (mm ³)			Tool wear (μm)			Tool life $\times 1,000$ (mm)		
	R1	R2	R3	R1	R2	R3	R1	R2	R3
1	27.75	28.5	27.0	260	270	250	11.1	11.4	10.8
2	112.8	112.8	110.4	250	260	240	14.1	14.1	13.8
3	360	366	360	240	260	250	18.0	18.3	18.0
4	76.5	78	75	230	210	220	15.3	15.6	15.0
5	513.6	518.4	508.8	210	210	190	32.1	32.4	31.8
6	85.5	87.0	85.5	220	240	230	17.1	17.4	17.1
7	60	61.2	62.4	230	220	210	15.0	15.3	15.6
8	162	165	162	220	240	230	16.2	16.5	16.2
9	120	123	126	200	200	220	12.0	12.3	12.6
10	402	408	408	210	210	200	20.1	20.4	20.4
11	45	45.75	45	220	240	220	18.0	18.3	18.0
12	153.6	156	151.2	210	200	200	19.2	19.5	18.9
13	643.2	628.8	648	210	200	220	40.2	39.3	40.5
14	75	75	73.5	280	290	290	15.0	15.0	14.7
15	175.5	177	175.5	250	260	280	35.1	35.4	35.1
16	105	108	102	220	240	210	10.5	10.8	10.2
17	120	120	123	250	240	210	12.0	12.0	12.3
18	39.6	38.4	38.4	200	220	210	9.9	9.6	9.6

Table 5 Normalised experimental results for individual quality characteristics (rough machining)

Experiment no.	Volume of material removed			Tool wear			Tool life		
	R1	R2	R3	R1	R2	R3	R1	R2	R3
Ideal	1	1	1	1	1	1	1	1	1
1	0.00000	0.00000	0.00000	0.25000	0.22222	0.40000	0.03960	0.06061	0.03884
2	0.13819	0.14043	0.13430	0.37500	0.33333	0.50000	0.13861	0.15152	0.13592
3	0.53985	0.56222	0.53623	0.50000	0.33333	0.40000	0.26733	0.29293	0.27185
4	0.07921	0.08246	0.07730	0.62500	0.88889	0.70000	0.17822	0.20202	0.17476
5	0.78942	0.81609	0.77585	0.87500	0.88889	1.00000	0.73267	0.76768	0.71845
6	0.09383	0.09745	0.09420	0.75000	0.55556	0.60000	0.23762	0.26263	0.24272
7	0.05240	0.05447	0.05701	0.62500	0.77778	0.80000	0.16832	0.19192	0.19418
8	0.21813	0.22739	0.21739	0.75000	0.55556	0.60000	0.20792	0.23232	0.21359
9	0.14989	0.15742	0.15942	1.00000	1.00000	0.70000	0.06931	0.09091	0.09709
10	0.60809	0.63218	0.61353	0.87500	0.88889	0.90000	0.33664	0.36364	0.34952
11	0.02803	0.02874	0.02899	0.75000	0.55556	0.70000	0.26733	0.29293	0.27185
12	0.20449	0.21239	0.20000	0.87500	1.00000	0.90000	0.30693	0.33333	0.30097
13	1.00000	1.00000	1.00000	0.87500	1.00000	0.70000	1.00000	1.00000	1.00000
14	0.07677	0.07746	0.07488	0.00000	0.00000	0.00000	0.16832	0.18182	0.16505
15	0.24007	0.24738	0.23913	0.37500	0.33333	0.10000	0.83168	0.86869	0.82524
16	0.12552	0.13243	0.12077	0.75000	0.55556	0.80000	0.01980	0.04040	0.01942
17	0.14989	0.15242	0.15459	0.37500	0.55556	0.80000	0.06931	0.08081	0.08738
18	0.01925	0.01649	0.01836	1.00000	0.77778	0.80000	0.00000	0.00000	0.00000

Table 6 Grey relational coefficients and grades (rough machining)

Experiment no.	Volume of material removed			Tool wear			Tool life			Grey relational grade	Order
	R1	R2	R3	R1	R2	R3	R1	R2	R3		
1	0.33333	0.33333	0.33333	0.40000	0.39130	0.45455	0.34237	0.34737	0.34219	0.36420	17
2	0.36716	0.36776	0.36611	0.44444	0.42857	0.50000	0.36727	0.37079	0.36655	0.39763	16
3	0.52075	0.53317	0.51880	0.50000	0.42857	0.45455	0.40562	0.41423	0.40712	0.46476	9
4	0.35192	0.35272	0.35144	0.57143	0.81818	0.62500	0.37828	0.38821	0.37729	0.46794	8
5	0.70365	0.73109	0.69046	0.80000	0.81818	1.00000	0.65161	0.68276	0.63975	0.74639	2
6	0.35558	0.35649	0.35567	0.66667	0.52941	0.55556	0.39608	0.40408	0.39768	0.44636	14
7	0.34540	0.34590	0.34650	0.57143	0.69231	0.71429	0.37547	0.38224	0.38290	0.46183	10
8	0.39006	0.39289	0.38983	0.66667	0.52941	0.55556	0.38697	0.39442	0.38868	0.45494	11
9	0.37034	0.37242	0.37297	1.00000	1.00000	0.62500	0.34948	0.35484	0.35640	0.53350	5
10	0.56060	0.57616	0.56403	0.80000	0.81818	0.83333	0.42979	0.43999	0.43460	0.60630	3
11	0.39680	0.33984	0.33990	0.66667	0.52941	0.62500	0.40562	0.41423	0.40712	0.45194	12
12	0.38595	0.38832	0.38462	0.80000	1.00000	0.83333	0.41909	0.42857	0.41700	0.56188	4
13	1.00000	1.00000	1.00000	0.80000	1.00000	0.62500	1.00000	1.00000	1.00000	0.93611	1
14	0.35131	0.35148	0.35085	0.33333	0.33333	0.33333	0.37547	0.37931	0.37455	0.35366	18
15	0.39685	0.39916	0.39655	0.44444	0.42857	0.35714	0.74815	0.79200	0.74101	0.52265	6
16	0.36377	0.36561	0.36252	0.66667	0.52941	0.71429	0.33779	0.34256	0.33771	0.44670	13
17	0.37034	0.37104	0.37163	0.44444	0.52941	0.71429	0.34948	0.35231	0.35395	0.42855	15
18	0.33767	0.33704	0.33746	1.00000	0.69231	0.71429	0.33333	0.33333	0.33333	0.49097	7

higher normal stress on the cutting edge, which is an important factor that determines the feasibility of high speed machining.

5.1.1 Grey relational analysis—rough machining

Grey relational analysis is an effective method for analysis of many factors using fewer data. It can provide an efficient solution to the uncertainty in multi-input and discrete data problems to optimise the multi-response processes through the setting of process parameters. The data pre-processing is the first step in this method. Next, the grey relational coefficient is calculated from the normalised experimental data to express the relationship between the ideal (best) and the actual experimental data. Then, grey relational grade is computed by averaging the grey relational coefficients

corresponding to each process response. The overall evaluation of the multiple process responses is based on the grey relational grade. The optimum level of the process parameters is exactly the level corresponding to the highest grey relational grade. The experimental results and normalised experimental results for individual quality characteristics are shown in Tables 4 and 5.

From Table 6, it is observed that the experiment no.13 has the highest grey relational grade. The levels of parameters in the above experiment are volume of material removed ($640 \times 1,000 \text{ mm}^3$), tool wear ($210 \text{ }\mu\text{m}$) and tool life ($40 \times 1,000 \text{ mm}$). It is close to the best machining parameters. The optimum process parameters and their effects on selected output parameters can be found out. For cutting speed, the mean of grey relational grade at levels 1, 2 and 3 can be calculated by averaging the grey relational

Table 7 Significance of machining parameters

Machining parameters	Average grey relational grade by process parameters level (experimental layout shown in Table 2)			Significance of machining parameters
	Level 1	Level 2	Level 3	
Cutting speed (A)	0.47445	0.57885 ^a	0.46941	0.10944
Feed (B)	0.54718 ^a	0.47219	0.50335	0.07499
Depth of cut (C)	0.46146	0.59913 ^a	0.46212	0.13767
Width of cut (D)	0.42816	0.47529	0.61927 ^a	0.19110

^a Optimised level of parameters (rough machining)

Table 8 ANOVA results for grey relational grade (rough machining)

Parameter	DF	Sum of square	Mean square	F ratio	P value
Cutting speed (A)	2	0.04581	0.02290	2.61884	0.12694
Feed (B)	2	0.01703	0.00852	0.97382	0.41414
Depth of cut (C)	2	0.07545	0.03773	4.31400	0.04855
Width of cut (D)	2	0.11894	0.05947	6.80052	0.01587
Error	9	0.07871	0.00875		
Total	17	0.33594			

grades of the experiments (1–3:10–12), (4–6:13–15), (7–9:16–18), respectively. Similarly, the mean of grey relational grades of other machining parameters, i.e. feed, depth of cut and width of cut at different levels were calculated in the same manner. The mean of grey relational grades of all parameters at different levels and the difference between the maximum and minimum value of the grey relational grade of the machining parameters are shown in Table 7. The maximum and minimum values of the grey relational grade show the importance of individual parameter in rough machining. The importances of rough machining parameters are in the order width of cut, depth of cut, cutting speed and feed. The machining parameter levels for cutting speed are A1 A2 A3. The machining parameter levels for feed, depth of cut and width of cut are B1 B2 B3, C1 C2 C3 and D1 D2 D3, respectively. It is observed from Table 7 that the highest grey relational grade of each parameter shows the optimal level of parameters. The optimised parameters are noticed as A2 (cutting speed: level 2), B1 (feed: level 1), C2 (depth of cut: level 2), D3 (width of cut: level 3) for better performance in rough machining. The higher value of grey relational grade is near to the product quality. The optimised parameters are A2B1C2D3. The corresponding values are cutting speed 180 m/min, feed 0.5 mm/tooth, depth of cut 0.8 mm and width of cut 20 mm.

5.1.2 Analysis of variance—rough machining

The main aim of ANOVA is to apply a statistical method in order to identify the effect of individual factors. The impact of each factor on results can be determined very clearly using ANOVA. The effect of individual parameters on the entire process cannot be judged by Taguchi method whilst the percentage contribution of individual parameters can be well determined using ANOVA. MATHEMATICA software of ANOVA module was employed to investigate out the effect of process parameters, i.e. cutting speed, feed, depth of cut and width of cut on grey relational grade. From Table 8, it is observed that the *P* value of the parameters is less than 0.05 thus indicating that the input parameters, i.e. width of cut and depth of cut are significantly contributing towards machining performance. ANOVA also results in the same order of importance of the machining parameters

as, i.e. width of cut, depth of cut, cutting speed and feed. From Table 9, it is observed that the cell means in the ANOVA test also results in the same optimum parameter settings, i.e. cutting speed 180 m/min, feed 0.5 mm/tooth, depth of cut 0.8 mm and width of cut 20 mm. In summary, the results obtained from ANOVA are closely matching with the results of grey relational analysis.

5.1.3 Verification test

Once the optimal level of process parameters are identified, the verification of improvement of performance at optimum level is estimated by using the grey relational grade. The estimated grey relational grade $\hat{\alpha}$ is calculated as follows:

$$\hat{\alpha} = \alpha_m + \sum_{i=1}^q (\bar{\alpha} - \alpha_m) \quad (10)$$

where α_m is total mean of grey relational grade, $\bar{\alpha}$ is the mean of grey relational grade at optimal level and q is the number of parameters that significantly affect the performance characteristics. In final experiment for the validation, the improvement in machining performance with optimal process parameters is confirmed by increasing grey relational grade. This is shown in Table 10. The volume of material removed, tool wear and tool life are improved with optimised machining parameters.

5.2 Finish machining

Experimental investigations were performed to study the effects of cutting speed on surface integrity produced on

Table 9 Cell mean of parameters in ANOVA (rough machining)

Parameter	Level 1	Level 2	Level 3
Cutting speed (A)	0.47445	0.57885 ^a	0.46941
Feed (B)	0.54718 ^a	0.47219	0.50335
Depth of cut (C)	0.46146	0.59913 ^a	0.46212
Width of cut (D)	0.42816	0.47529	0.61927 ^a

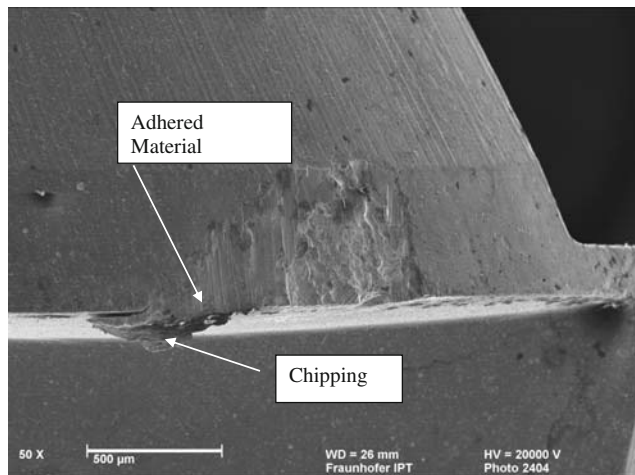
All cell mean=0.50757

^a Optimised level of parameters

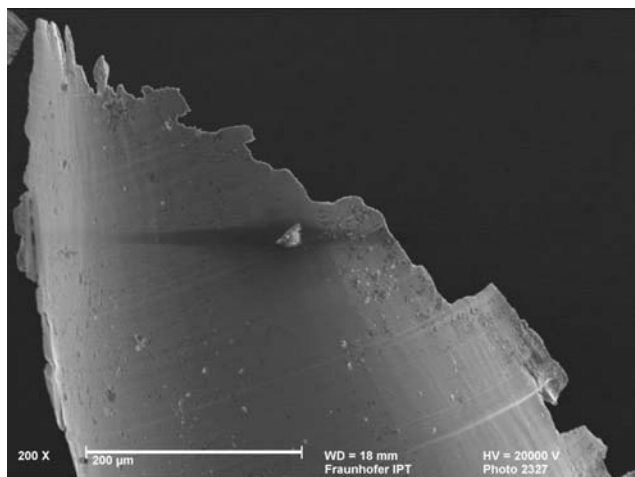
Table 10 Improvements in grey relational grade with optimised machining parameters (rough machining)

Setting level	Initial data $A_2B_2C_2D_3$	Optimal machining parameters	
		Prediction $A_2B_1C_2D_3$	Experiment $A_2B_1C_2D_3$
Volume of material removed $\times 1,000$ (mm ³)	513.6		640.0
Tool wear (μm)	205.0		210.0
Tool life $\times 1,000$ (mm)	32.1		40.0
Improvement in grey relational grade=0.18972	0.74639	0.71083	0.93611

difficult-to-machine materials by Kishawy and Elbestawi [40,41]. It was observed that a low value of surface roughness was always obtained for a range of the cutting speeds employed. In order to obtain a surface of minimum residual stresses, geometrical defects and good surface finish, feed and depth of cut are the factors to be considered for optimisation [41]. This has been confirmed by investigation study performed by Oishi [42]. It is observed



(a) SEM picture of cutting tool insert



(b) SEM picture of chip

Fig. 6 Tool wear and chip formation at cutting speed 204 m/min, feed 0.2 mm/tooth, depth of cut 0.2 mm and width of cut 0.2 mm. **a** SEM picture of cutting tool insert. **b** SEM picture of chip

through experimental investigations for high speed machining of hardened steel that the tool life is longer for a feed range of 0.05–0.1 mm/tooth reportably giving the best results [43]. However, the tool life is shortened at the feed 0.05 mm/tooth investigated by Koshy et al. [23]. The reason is that the inefficient material removal is obtained by rubbing rather than efficient cutting, as a result of insufficient undeformed chip thickness. Experiments were performed for a feed (fz) in the range of 0.05–0.2 mm/tooth, for cutting speeds (Vc) 100 to 204 m/min with depth of cut (ap) as 0.05–0.2 mm and width of cuts (ae) in the range 0.1 to 0.4 mm. The tool wear pattern is shown in Fig. 6. It is observed from SEM pictures that chipping and adhesion are prominent tool wear failures. The effects of varying feed on tool wear, tool life and surface finish on various cutting conditions are shown Figs. 7, 8 and 9. It is observed that the best results are obtained for the range of feed 0.1–0.2 mm/tooth. From Fig. 8, it is observed that tool life is maximum, i.e. $1,150 \times 1,000$ mm at cutting speed (Vc) 204 m/min corresponding to process parameters feed (fz) 0.2 mm/tooth, depth of cut (ap) 0.2 mm and width of cut (ae) 0.2 mm. The measured surface finish (Ra) is in the range of 0.4–0.52 μm .

5.2.1 Grey relational analysis—finish machining

The procedure is same for finish machining to optimise process parameters as followed in rough machining. The experimental layout is shown in Table 2. The machining parameters are shown in Table 11. The normalised experimental results for individual quality characteristics are calculated from the experimental results (Table 12) by Eqs. 1 and 2. These are shown in Table 13. The grey relational coefficients and grey relational grade are calculated by the Eqs. 3 and 4, respectively, and shown in Table 14. It is observed from Table 14 that the experiment no. 8 has the highest grey relational grade and the value is 0.93464. From Table 12, the average values for the output parameters, i.e. surface finish, tool wear and tool life are 0.42 μm , 95 μm , $914.7 \times 1,000$ mm, respectively, corresponding to the experiment no. 8. The optimum process parameters and their effects on selected output are tabulated in Table 15. The mean of grey relational grade at

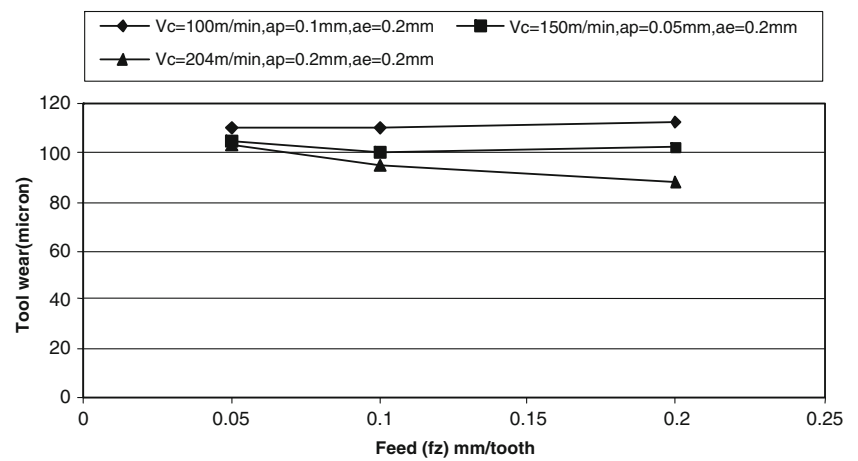
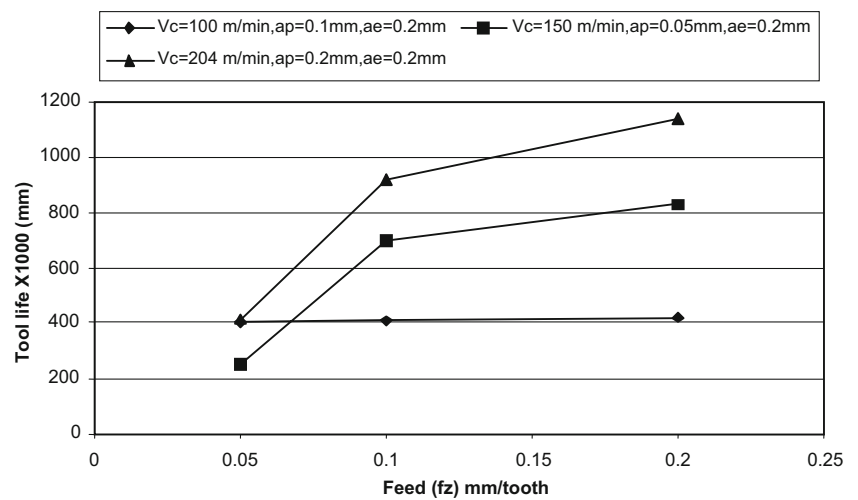
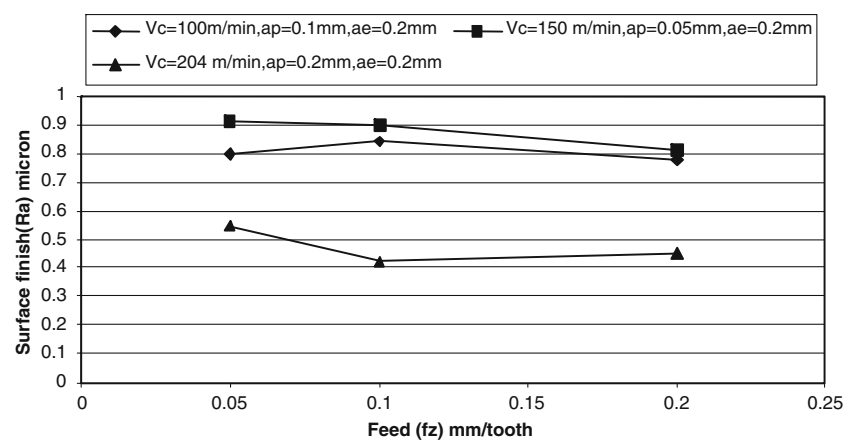
Fig. 7 Tool wear vs feed (finish machining)**Fig. 8** Tool life vs feed (finish machining)**Fig. 9** Surface finish vs feed (finish machining)

Table 11 Machining parameters and their levels (finish machining)

Machining parameters	Unit	Level 1	Level 2	Level 3
Cutting speed (A)	m/min	100	150	204
Feed (B)	mm/tooth	0.05	0.1	0.2
Depth of cut (C)	mm	0.05	0.1	0.2
Width of cut (D)	mm	0.1	0.2	0.4

level 1 for the cutting speed is calculated by averaging the grey relational grades of the experiments (1–3:10–12) from the Tables 2 and 14. The average of the grey relational grades 0.37939, 0.40931, 0.47889, 0.39571, 0.48339 and 0.41611 is 0.42713. Similarly, the mean of grey relational grade at levels 2 and 3 can be calculated by averaging the grey relational grades of the experiments (4–6:13–15) and (7–9:16–18), respectively. The averages of the grey relational grades are 0.46178 and 0.64812. The mean of grey relational grades of other machining parameters, i.e. feed, depth of cut and width of cut at different levels are calculated in the same manner. The mean of grey relational grades of all parameters at different levels and the difference between the maximum and minimum value of the grey relational grade of the machining parameters are shown in Table 15. The maximum and minimum values of the grey relational grade show the importance of individual parameter in finish machining. The importances of finish machining parameters are in the order cutting speed and feed. The machining parameter levels for cutting speed are

A1 A2 A3. The machining parameter levels for feed, depth of cut and width of cut are B1 B2 B3, C1 C2 C3 and D1 D2 D3, respectively. It is observed from Table 15 that the highest grey relational grade of each parameter shows the optimal level of parameters. The optimised parameters are noticed as A3 (cutting speed: level 3), B3 (feed: level 3), C3 (depth of cut: level 3) and D2 (width of cut: level 2) for better performance in finish machining. The higher value of grey relational grade is near to the product quality. The optimised parameters are A3B3C3D2. The corresponding values are cutting speed 204 m/min, feed 0.2 mm/tooth, depth of cut 0.2 mm and width of cut 0.2 mm.

5.2.2 Analysis of variance—finish machining

For finish machining, the same procedure adopted in rough machining is followed in order to analyse the process parameters affecting significantly the performance characteristics, i.e. cutting speed, feed, depth of cut and width of cut on grey relational grade. From Table 16, it is observed that the *P* value of the parameters is less than 0.05 thus indicating that the input parameter, i.e. cutting speed is significantly contributing towards machining performance. ANOVA also results in the same order of importance of the machining parameters as cutting speed, feed, width of cut and depth of cut. From Table 17, it is observed that the cell means in the ANOVA test also results in the same optimum parameter setting, i.e. cutting speed 204 m/min, feed 0.2 mm/tooth, depth of cut 0.2 mm and width of cut

Table 12 Experimental results for surface finish, tool wear and tool life (finish machining)

Experiment no.	Surface finish (μm)			Tool wear (μm)			Tool life $\times 1,000$ (mm)		
	R1	R2	R3	R1	R2	R3	R1	R2	R3
1	0.90	0.92	0.95	110	110	105	290.20	280.00	270.03
2	0.80	0.85	0.86	120	110	100	420.00	410.03	400.10
3	0.91	0.92	0.94	90	90	110	310.10	290.20	305.03
4	0.90	0.90	0.92	105	100	110	250.08	250.25	252.00
5	0.92	0.93	0.70	110	95	85	300.13	310.30	302.10
6	0.80	0.70	0.90	100	90	110	448.00	409.50	406.35
7	0.61	0.62	0.70	110	95	95	437.50	446.25	439.25
8	0.41	0.42	0.43	90	85	110	910.00	918.75	915.25
9	0.85	0.90	0.75	95	100	100	782.25	784.00	780.50
10	0.91	0.82	0.92	100	110	110	250.10	252.00	249.40
11	0.95	0.82	0.92	95	90	95	320.10	321.12	322.00
12	0.81	0.79	0.75	110	120	105	420.00	427.00	418.25
13	0.95	0.96	0.85	100	125	90	300.13	320.25	315.00
14	0.90	0.95	0.96	105	100	120	450.10	410.40	470.10
15	0.76	0.82	0.85	100	100	105	822.15	830.03	841.05
16	0.61	0.55	0.50	105	100	105	420.00	410.03	405.13
17	0.85	0.81	0.83	110	105	110	800.10	791.35	780.15
18	0.52	0.51	0.49	90	95	100	890.05	885.15	900.03

Table 13 Normalised experimental results for individual quality characteristics (finish machining)

Experiment no.	Surface finish			Tool wear			Tool life		
	R1	R2	R3	R1	R2	R3	R1	R2	R3
Ideal	1	1	1	1	1	1	1	1	1
1	0.09259	0.07407	0.01887	0.33333	0.37500	0.42857	0.06080	0.04450	0.03098
2	0.27778	0.20370	0.18868	0.00000	0.37500	0.57143	0.25749	0.23901	0.22633
3	0.07407	0.07407	0.03774	1.00000	0.87500	0.28571	0.09095	0.05976	0.08355
4	0.09259	0.11111	0.07547	0.50000	0.62500	0.28571	0.00000	0.00000	0.00391
5	0.05556	0.05556	0.49057	0.33333	0.75000	1.00000	0.07584	0.08983	0.07915
6	0.27778	0.48148	0.11321	0.66667	0.87500	0.28571	0.29992	0.23822	0.23571
7	0.62963	0.62963	0.49057	0.33333	0.75000	0.71429	0.28400	0.29319	0.28512
8	1.00000	1.00000	1.00000	1.00000	1.00000	0.28571	1.00000	1.00000	1.00000
9	0.18519	0.11111	0.39623	0.83333	0.62500	0.57143	0.80642	0.79843	0.79763
10	0.07407	0.25926	0.07547	0.66667	0.37500	0.28571	0.00003	0.00262	0.00000
11	0.00000	0.25926	0.07547	0.83333	0.87500	0.71429	0.10610	0.10601	0.10903
12	0.25926	0.31481	0.39623	0.33333	0.12500	0.42857	0.25749	0.26440	0.25359
13	0.00000	0.00000	0.20755	0.66667	0.00000	0.85714	0.07584	0.10471	0.09852
14	0.09259	0.01852	0.00000	0.50000	0.62500	0.00000	0.30310	0.23957	0.33146
15	0.35185	0.25926	0.20755	0.66667	0.62500	0.42857	0.86688	0.86729	0.88856
16	0.62963	0.75926	0.86793	0.50000	0.62500	0.42857	0.25749	0.23901	0.23388
17	0.18519	0.27778	0.24528	0.33333	0.50000	0.28571	0.83346	0.80942	0.79710
18	0.79630	0.83333	0.88679	1.00000	0.75000	0.57143	0.96977	0.94974	0.97714

Table 14 Grey relational coefficients and grades (finish machining)

Experiment no.	Surface finish			Tool wear			Tool life			Grey relational grade	Order
	R1	R2	R3	R1	R2	R3	R1	R2	R3		
1	0.35526	0.35065	0.33758	0.42857	0.44444	0.46667	0.34741	0.34353	0.34036	0.37939	18
2	0.40909	0.38571	0.38130	0.33333	0.44444	0.53846	0.40241	0.39652	0.39257	0.40931	14
3	0.35065	0.35065	0.34194	1.00000	0.80000	0.41176	0.35485	0.34717	0.35299	0.47889	10
4	0.35526	0.35999	0.35099	0.50000	0.57143	0.41176	0.33333	0.33333	0.33420	0.39448	17
5	0.34615	0.34615	0.49533	0.42857	0.66667	1.00000	0.35109	0.35457	0.35190	0.48227	9
6	0.40909	0.49091	0.36054	0.60000	0.80000	0.41177	0.41664	0.39627	0.39548	0.47563	11
7	0.57447	0.57447	0.49533	0.42857	0.66667	0.63636	0.41119	0.41432	0.41157	0.51255	7
8	1.00000	1.00000	1.00000	1.00000	1.00000	0.41177	1.00000	1.00000	1.00000	0.93464	1
9	0.38028	0.35999	0.45299	0.74999	0.57143	0.53846	0.72089	0.71269	0.71187	0.57762	4
10	0.35065	0.40299	0.35099	0.60000	0.44444	0.41177	0.33334	0.33392	0.33333	0.39571	16
11	0.33333	0.40299	0.35099	0.74999	0.80000	0.63636	0.35871	0.35868	0.35946	0.48339	8
12	0.40299	0.42188	0.45299	0.42857	0.36364	0.46667	0.40241	0.40466	0.40115	0.41611	13
13	0.33333	0.33333	0.38686	0.60000	0.33333	0.77778	0.35109	0.35835	0.35677	0.42565	12
14	0.35526	0.33750	0.33333	0.50000	0.57143	0.33333	0.41775	0.39669	0.42788	0.40813	15
15	0.43548	0.40299	0.38686	0.60000	0.57143	0.46667	0.78974	0.79025	0.81775	0.58457	3
16	0.57447	0.67500	0.79105	0.50000	0.57143	0.46667	0.40241	0.39652	0.39491	0.53027	5
17	0.38028	0.40909	0.39850	0.42857	0.50000	0.41177	0.75015	0.72403	0.71134	0.52375	6
18	0.71053	0.74999	0.81539	1.00000	0.66667	0.53846	0.94299	0.90866	0.95628	0.80989	2

Table 15 Significance of machining parameters

Machining parameters	Average grey relational grade by process parameters level (experimental layout shown in Table 2)			Significance of machining parameters Max–min
	Level 1	Level 2	Level 3	
Cutting speed (A)	0.42713	0.46178	0.64812 ^a	0.22099
Feed (B)	0.43967	0.54025	0.55712 ^a	0.11745
Depth of cut (C)	0.49053	0.50930	0.53721 ^a	0.04669
Width of cut (D)	0.51150	0.54489 ^a	0.48065	0.06424

^a Optimised level of parameters (finish machining)**Table 16** ANOVA results for grey relational grade (finish machining)

Parameter	DF	Sum of square	Mean square	F ratio	P value
Cutting speed (A)	2	0.16951	0.08476	5.98153	0.02226
Feed (B)	2	0.04839	0.02419	1.70740	0.23516
Depth of cut (C)	2	0.00662	0.00331	0.23363	0.79631
Width of cut (D)	2	0.01239	0.00620	0.43722	0.65885
Error	9	0.12573	0.01417		
Total	17	0.36443			

Table 17 Cell mean of parameters in ANOVA (finish machining)

Parameter	Level 1	Level 2	Level 3
Cutting speed (A)	0.42713	0.46178	0.64812 ^a
Feed (B)	0.43967	0.54025	0.55712 ^a
Depth of cut (C)	0.49053	0.50930	0.53721 ^a
Width of cut (D)	0.51150	0.54489 ^a	0.48065

All cell mean=0.51235

^a Optimised level of parameters**Table 18** Improvements in grey relational grade with optimised machining parameters (finish machining)

Setting level	Initial data A ₃ B ₂ C ₃ D ₂	Optimal machining parameters	
		Prediction A ₃ B ₃ C ₃ D ₂	Experiment A ₃ B ₃ C ₃ D ₂
Surface finish (μm)	0.42		0.45
Tool wear (μm)	95		88
Tool life ×1,000 (mm)	914.7		1137.5
Improvement in grey relational grade=0.02235	0.93464	0.57183	0.95699

0.2 mm. In summary, the results obtained from ANOVA are closely matching with the results of grey relational analysis.

5.2.3 Verification test

When the optimal levels of process parameters are identified, the verification for performance improvement at optimum level is estimated by using the grey relational grade. The estimated grey relational grade ($\hat{\alpha}$) is calculated by the Eq. 10. For the validation, the improvement in machining performance with optimal process parameters is confirmed by increased grey relational grade. This is shown in Table 18. The surface finish, tool wear and tool life are improved with optimised machining parameters.

6 Conclusions

Grey relational analysis is the effective and efficient method for optimising multi response process parameters. The process parameters for end milling whilst hard machining of hardened steel are optimised with L_{18} orthogonal array and grey relational analysis. The results are compared with ANOVA. It has been observed that the width of cut and depth of cut are the most influencing parameters in the case of rough machining corresponding to the quality characteristics of tool life, tool wear and volume of material removed. For finish machining, the cutting speed is the most influencing parameter corresponding to the quality characteristics of tool life, tool wear and surface finish. The causes of tool wear are chipping and adhesion. The optimum parameters for rough machining are at cutting speed 180 m/min, feed 0.5 mm/tooth, depth of cut 0.8 mm and width of cut 20 mm whereas for finish machining, the optimum parameters are at cutting speed 204 m/min, feed 0.2 mm/tooth, depth of cut 0.2 mm and width of cut 0.2 mm.

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Appendix

Numerical example:

The normalised values corresponding to experimental no. 3 in Table 5

The normalised value for volume of material removed (R1) is calculated using Eq. 1

$$(360 - 27.75)/(643.2 - 27.75) = 0.53985$$

Similarly, R2 can be calculated as

$$(366 - 28.5)/(628.8 - 28.5) = 0.56222.$$

R3 can be calculated as

$$(360 - 27)/(648 - 27) = 0.53623.$$

The normalised value for tool wear (R1) is calculated using Eq. 2

$$(280 - 240)/(280 - 200) = 0.50000.$$

Similarly, R2 can be calculated as

$$(290 - 260)/(290 - 200) = 0.33333.$$

R3 can be calculated as

$$(290 - 250)/(290 - 190) = 0.40000.$$

The normalised value for tool life (R1) is calculated using Eq. 1

$$(18 - 9.9)/(40.2 - 9.9) = 0.26733.$$

Similarly, R2 can be calculated as

$$(18.3 - 9.6)/(39.3 - 9.6) = 0.29293.$$

R3 can be calculated as

$$(18 - 9.6)/(40.5 - 9.6) = 0.27185.$$

The grey relational coefficient corresponding to experimental no. 3 in Table 6

The grey relational coefficient for volume of material removed (R1) is calculated using Eq. 3

$$\begin{aligned} & [(1 - 1) + 0.5 \times (1 - 0)] / [(1 - 0.53985) + 0.5 \times (1 - 0)] \\ & = 0.52075. \end{aligned} \quad (11)$$

Similarly, R2 can be calculated as

$$\begin{aligned} & [(1 - 1) + 0.5 \times (1 - 0)] / [(1 - 0.56222) + 0.5 \times (1 - 0)] \\ & = 0.53317. \end{aligned} \quad (12)$$

R3 can be calculated as

$$\begin{aligned} & [(1 - 1) + 0.5 \times (1 - 0)] / [(1 - 0.53623) + 0.5 \times (1 - 0)] \\ & = 0.51880. \end{aligned} \quad (13)$$

The grey relational coefficient for tool wear (R1) is calculated using Eq. 3

$$\begin{aligned} & [(1 - 1) + 0.5 \times (1 - 0)] / [(1 - 0.5) + 0.5 \times (1 - 0)] \\ & = 0.50000. \end{aligned} \quad (14)$$

Similarly, R2 can be calculated as

$$\begin{aligned} & [(1 - 1) + 0.5 \times (1 - 0)] / [(1 - 0.33333) + 0.5 \times (1 - 0)] \\ & = 0.42857. \end{aligned} \quad (15)$$

R3 can be calculated as

$$\begin{aligned} & [(1 - 1) + 0.5 \times (1 - 0)] / [(1 - 0.4) + 0.5 \times (1 - 0)] \\ & = 0.45455. \end{aligned} \quad (16)$$

The grey relational coefficient for tool life (R1) is calculated using Eq. 3

$$\begin{aligned} & [(1 - 1) + 0.5 \times (1 - 0)] / [(1 - 0.26733) + 0.5 \times (1 - 0)] \\ & = 0.40562. \end{aligned} \quad (17)$$

Similarly, R2 can be calculated as

$$\begin{aligned} & [(1 - 1) + 0.5 \times (1 - 0)] / [(1 - 0.29293) + 0.5 \times (1 - 0)] \\ & = 0.41423. \end{aligned} \quad (18)$$

R3 can be calculated as

$$\begin{aligned} & [(1 - 1) + 0.5 \times (1 - 0)] / [(1 - 0.27185) + 0.5 \times (1 - 0)] \\ & = 0.40712. \end{aligned} \quad (19)$$

The grey relational grade is calculated using the average of Eqs. 11 to 19

$$\begin{aligned} & = [(0.52075 + 0.53317 + 0.51880 + 0.50000 + 0.42857 \\ & \quad + 0.45455 + 0.40562 + 0.41423 + 0.40712)] / 9 \\ & = 0.46476. \end{aligned}$$

The grey relational grade for experiment no. 3 is 0.46476.

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